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Eliminating spiral waves and spatiotemporal chaos using feedback signal

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Abstract. Spiral waves and spatiotemporal chaos are sometimes harmful and should be controlled. In this letter we present a feedback scheme to eliminate them. We first collect feedback signals at a certain time t_0. Then wait for the system at the excitable position to enter the recovering state. When the time comes, the feedback signals are added. This scheme has two advantages. Firstly, the tip can be eliminated together with the body of spiral wave. Secondly, the injected feedback signals can be very weak and the duration can be very short so that the original system is nearly not to be affected, which is important for practical applications.

PACS. 05.70.Ln Nonequilibrium and irreversible thermodynamics – 05.45.Gg Control of chaos, applications of chaos

1 Introduction

Spiral waves are ubiquitous in excitable and self-oscillating media, such as chemical systems with reaction,the oxidation of CO on platinum, and cardiac muscles, etc. [1]. Sometimes such a phenomenon presents undesirable results. For example, spiral waves in cardiac muscles may result in tachycardia. Repetitious breakup of spiral waves can lead to spatiotemporal chaos, which is believed to be a possible mechanism of ventricular fibrillation (VF). The VF and sudden cardiac death are the leading causes of cardiovascular mortality in industrialized countries [2,3]. The current therapy of the VF is electrical shocking with high voltage (about 600 Volts), which is painful and may create scars promoting future episodes of VF. To develop alternative therapies, it is necessary to desire schemes for controlling and eliminating spatiotemporal chaos through low-amplitude signals.

There have been several methods to control and eliminate spiral waves and spatiotemporal chaos [4–12]. For example, Sakguchi, et al. [4] suggested a method where the spiral wave is eliminated by periodic force in the Aliev-Panfilov model. In reference [5], the authors use weak spatial perturbations to eliminate spatiotemporal chaos and spiral waves. Suppressing arrhythmias in cardiac ionic models by overdrive pacing is reported in reference [10]. Controlling spiral waves and spatiotemporal chaos by a sine wave is suggested in reference [11]. In reference [12] spiral waves and spatiotemporal chaos are controlled and suppressed by the traveling waves excited by local perturbation. In this letter we propose a feedback scheme to eliminate spiral waves and spatiotemporal chaos. The strategy is as follows: the feedback signals are recorded at a certain time t_0, then injected into the system when the system at excitable position enters the recovery state. With appropriate intensive parameter and injecting duration spiral waves and spatiotemporal chaos can be successfully eliminated. There are two advantages with the method. Firstly, different from some methods with external force where the core of spiral wave is usually forced to move towards the boundary and to disappear there, in our scheme the tip can be rapidly eliminated together with the body of spiral wave. Secondly, the feedback may be very weak and the duration can be very short, so the original system can be little affected and harmed.

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2 The model

We first consider the FizHugh-Nagumo(FHN) model [13,14]. The model is generic for excitable systems. It can be applied to a variety of systems, and able to reproduce many qualitative characteristics of electrical impulses along nerve and cardiac fibers, such as the existence of an excitation threshold, relative and absolute refractory periods, and the generation of pulse trains under the action of external currents. The FHN model with a feedback term is described by the following equations:

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} f(u,v) + \nabla^2 u + \frac{A}{\varepsilon} I_{\text{feedback}} \tag{1}$$

$$\frac{\partial v}{\partial t} = g(u,v) \tag{2}$$

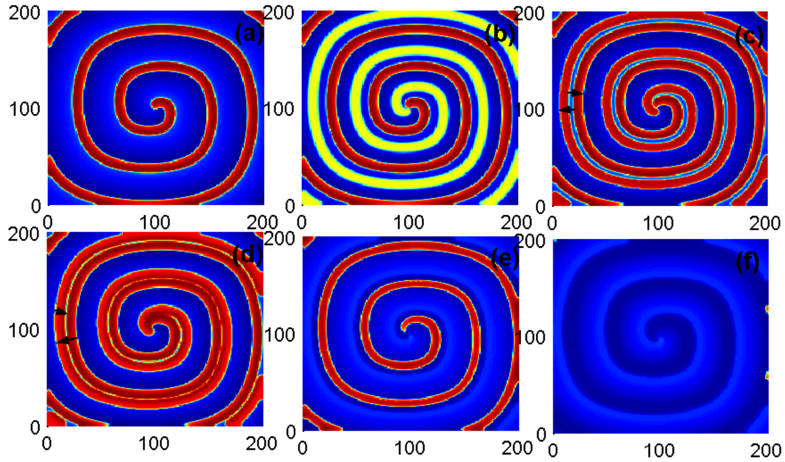

Fig. 1. Elimination of the spiral wave in the FHN system, here $t_1 = 86$, $t_2 = 88$, and $A = 0.3$. (a) $t = 86$, the spiral wave before the feedback signals are injected; (b) $t = 87$, at which the feedback signals are injected; (c) $t = 99$, an anti-spiral wave is excited by the feedback in the recovery positions of the original spiral wave; (d) $t = 105$, the original spiral wave begins to meet with the anti-spiral wave; (e) $t = 125$, the excited states start to diminish gradually; (f) $t = 145$, the spiral wave has been completely eliminated.

where $f(u, v) = -v - u(u-a)(u-1)$ and $g(u, v) = -\gamma v + \beta u - \delta$; u and v describe the membrane Voltage (fast variable) and the v-gate (slow variable), respectively; $\frac{A}{\varepsilon} I_{\text{feedback}}$ is the feedback term; ε is the excitability; A is an intensity parameter; a represents the threshold for excitation; β, γ and δ are parameters controlling the rest state and dynamics. By truncating a travel wave, a spiral wave can be formed in the absence of the feedback.

We consider also an activator controller two-variable reaction-diffusion model for catalytic CO oxidation on Pt (110) [15]. Its equation with a feedback term is as follows:

$$\frac{\partial u}{\partial t} = -\frac{1}{\varepsilon} u(u-1)(u - \frac{v+b}{a}) + \nabla^2 u + \frac{A}{\varepsilon} I_{\text{feedback}} \quad (3)$$

$$\frac{\partial v}{\partial t} = f(u) - v. \quad (4)$$

Here u and v describe the activator and the inhibitor variables, respectively. For the standard Barkley model, $f(u)$ is taken in the continuous form, $f(u) = u$. If $f(u)$ is discrete, we name the model the discrete Barkley. In the discrete Barkley model, $f(u) = 0$ when $0 \le u < 1/3$; $f(u) = 1 - 6.75u(u-1)^2$ when $1/3 \le u \le 1$; and $f(u) = 1$ when $u > 1$. In this letter we fix $a = 0.84$ and $b = 0.07$. In the absence of the feedback, spiral waves can be observed in the standard form. For the discrete form, the spatiotemporal dynamics is observed by varying ε. In the range, $0.01 < \varepsilon < 0.06$, suitable initial conditions lead to steady rotating spiral waves. At $\varepsilon = 0.06$, the spiral waves undergo a transition from steady rotation to meandering. When $\varepsilon > 0.07$, spiral waves will break up and the system will quickly fall into a turbulence state.

3 Eliminating spiral waves by feedback signal

The detailed scheme is as follows: the feedback signal, $I_{\text{feedback}} = u(t_0)$, is a function of space coordinates and is recorded at a certain time t_0. When the positions being in excited states at t_0 start exhibiting recovery ones, I_{feedback} is injected into the system from t_1 to t_2 (namely, $t_1 < t < t_2$). I_{feedback} is described as follows:

$$I_{\text{feedback}} = \begin{cases} u(t_0), & t_1 < t < t_2 \\ 0, & t \le t_1 \quad \text{or} \quad t \ge t_2. \end{cases} \quad (5)$$

For proper t_1 and t_2, spiral waves can be rapidly eliminated.

Firstly, we investigate the FHN model with no-flux boundary with the split-operator algorithm. The corresponding parameters are $a = 0.03$, $\gamma = 1.0$, $\beta = 2.0$, $\delta = 0$, and $\varepsilon = 0.005$. The time step and the grid spacing are separately equal to 0.0025 and 0.1. The number of space grids is 200×200. A spiral wave can be generated by truncating a travelling wave whose exited period is about 138 time units (1 time unit is equal to 1 time step, and equal to 1/138 of the excited period), where 34 time units corresponds excited states, and 104 time units is recovery states. If we take $t_0 = 10$ time units, then the excited locations at $t_0 = 10$ time units start being in recovery states from the 44 to the 148 time units within the first excited period after t_0. Spiral waves can be successfully eliminated by taking appropriately t_1 and t_2 in the time range from 44 time units to 148 time units. Figure 1 shows a representative eliminated progress of a spiral wave with the amplitude parameter $A = 0.3$ and the feedback duration of only 1 time unit, where the feedback signals are injected only at $t = 87$ time units. It is easily found that an anti-spiral wave, which travels towards the core, appears in the spatial domains of being in recovery states, and encounters the original spiral wave. From the characters of excited waves, as a result of the encounter, the excited states will disappear. In the progress, the tip of the spiral wave can be rapidly removed in its original positions and together with the body. So the elimination is achieved within 59 time units after that the feedback is injected. It is less than an excited period. This speedy character is important in many real applications. Figure 2 gives the ranges of A and $t_1$under the different durations of the feedback, in which spiral waves can be successfully eliminated. For the amplitude parameter A to eliminate spiral waves, a maximum A_{max} and a minimum value A_{min} can be observed (See Figs. 2a–2c). The existence of A_{min} is due to the impossibility of generating anti-spiral waves or

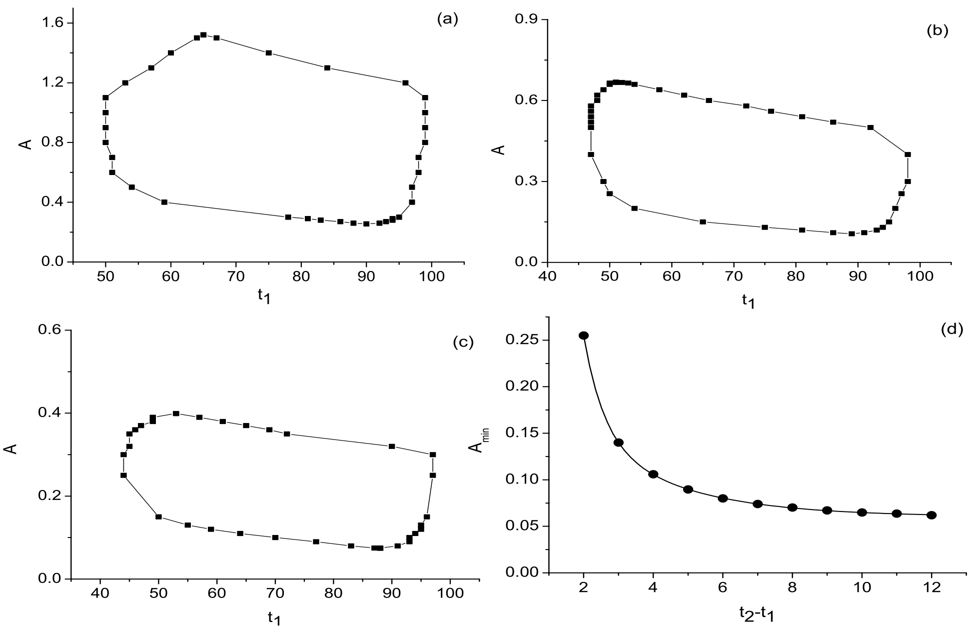

Fig. 2. The ranges of A and t_1 to eliminate spiral waves under the different durations of the feedback. (a) $t_1 < t < t_1 + 2$; (b) $t_1 < t < t_1 + 4$; (c) $t_1 < t < t_1 + 7$. Spiral waves can be eliminated when A and t_1 are taken in the domains enclosed by the curves and on the curves in (a–c). (d) The durations of the feedback $t_2 - t_1$ vs. the optional minimum amplitude A_{min}, the numeric results are depicted by the dots, and the fitting by exponential decay of second order are described by the line. The fitting function reads $A_{min} = a_1 e^{-(t_2-t_1)/b_1} + a_2 e^{-(t_2-t_1)/b_2} + A_0$, where $A_0 = 0.06033$, $a_1 = 0.18649$, $b_1 = 2.68215$, $a_2 = 3.38817$, $b_2 = 0.57756$.

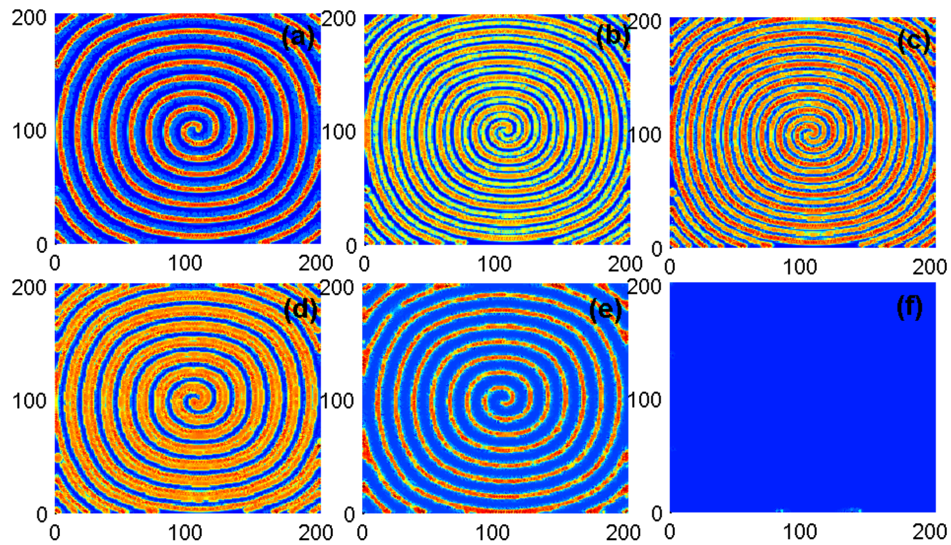

Fig. 3. The elimination of spiral waves in the standard Barkley model by feedback method, where $t_1 = 22$, $t_2 = 26$, and $A = 0.165$. (a) $t = 22$, the spiral wave before the feedback is introduced; (b) $t = 25$, the feedback has been injected; (c) $t = 27$, a anti-spiral wave is generated; (d) $t = 31$, the anti-spiral wave begin to meet with the original spiral wave; (e) $t = 39$, the excited domains are gradually diminishing; (f) $t = 53$, spatiotemporal chaos is completely eliminated.

the difficulty in abruptly increasing v value by small feedback signals. The appearance of A_{max} should be related to the sharp diminishment of u when the recovery parts of the feedback signal is injected into the positions being in the exited states. A_{max} and A_{min} will diminish with the increasing of $t_2 - t_1$. $A_{max} = 1.52$, $A_{min} = 0.255$ when $t_1 < t < t_1 + 2$; $A_{max} = 0.668$, $A_{min} = 0.106$ when $t_1 < t < t_1 + 4$; $A_{max} = 0.399$, $A_{min} = 0.074$ when $t_1 < t < t_1 + 7$. Figure 2d shows the relation of A_{min} and $t_2 - t_1$, which can be accurately fitted by exponential decay of second order. It is easily seen that the relation should be exponential decay, which means that A_{min} may be very small if the standing duration of the feedback is very long. (It is important in real applications that spiral wave is eliminated by weak feedback.) For the intermediate feedback intensity between A_{min} and A_{max}, the range of t_1 to eliminate spiral wave may be very wide. We also notice that the scheme works within an arbitrary excited period after t_0.

In order to see the universality of our scheme, the Barkley model is also considered. In the numeric simulation, the number of space grids is 202×202, and no-flux boundary is used. The semi-implicit form for local dynamics is adopted, and the five-point finite-difference Laplacian formula for the diffusion is considered. Feedback signal is recorded at $t_0 = 10$ time units. Figure 3 exhibits the progress of eliminating spiral wave with $A = 0.156$, $t_2 - t_1 = 3$ time units, and $t_1 = 22$ time units.

Here we will give a scientific insight into why the method works by the characters of excitable waves. There exist three possible wave patterns in a travelling wave or one-dimensional excitable system subjected to

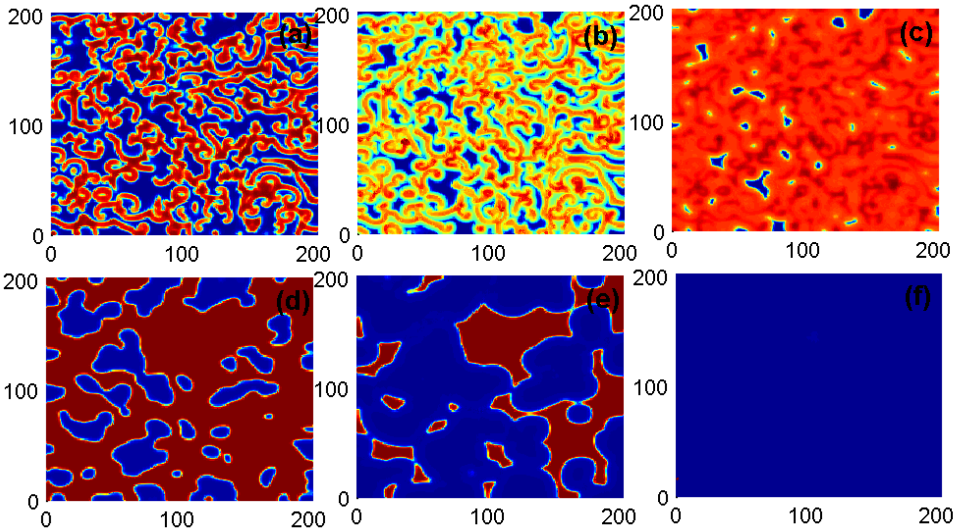

Fig. 4. Elimination of spatiotemporal chaos in the discrete Barkley model by feedback method, where $t_1 = 22$, $t_2 = 25$, and $A = 0.625$. (a) $t = 22$, the spatiotemporal chaos before the feedback is introduced; (b) $t = 24$, the feedback has been injected; (c) $t = 32$; (d) $t = 66$; (e) $t = 104$; (f) $t = 211$, spatiotemporal chaos is completely eliminated.

perturbation stimulation after excited states, namely, bidirectional, unidirectional and decaying propagation. The corresponding regions of unidirectional and decaying propagation are respectively called "vulnerable window(VW)" and "absolute refractory window". VW is a well-known classical notion for quasi one-dimensional excitable waves and ensures many characters of the excitable media [16–18], such as the death of two travelling waves encountering vis-a-vis with each other. If a stimulation is introduced in the "absolute refractory window", the original wave is little affected and no additional wave or pulse forms. If a perturbation exists in the "vulnerable window", an anti-directional wave or pulse, which propagates in the opposite direction to the original wave or pulse, may form. A pair of bidirectional waves or pulses can be generated when a perturbation lies in the window of bidirectional propagation. Similarly, there exist also the windows in the evolution of spiral wave. Due to the presence of "absolute refractory window", two waves propagating vis-a-vis will disappear when they encounter with each other. In our method the excitable state of the feedback recorded at a former time t_0 is injected into the system in "vulnerable window" of the original spiral waves and a new anti-directional spiral waves can be generated. Two spiral waves meet with each other after a transitory time and at last they will disappear simultaneously. If the excitable parts of the recorded signal is injected into the system in "absolute refractory window", the stimulation does not work and the system is little affected. For the window of bidirectional propagation, a new pair of spiral waves form, at last the anti-directional one will die together with the original spiral wave and the another of the pair will be developed as a new spiral wave in the system. It is easy to understood that the choice of the control time is intimately related to the "vulnerable window".

4 Eliminating spatiotemporal chaos by feedback signal

Repetitious breakup of spiral waves can lead to spatiotemporal chaos, which can be regarded as being composed of many undeveloped spiral waves. It is possible to eliminate spatiotemporal chaos by the above scheme. Here we use the discrete Barkley model. We take $\varepsilon = 0.08$ under which spontaneous breakup of spiral waves into spatiotemporal chaos can happen. The number of space grids is 202×202, and no-flux boundary is used. The semi-implicit form for local dynamics is adopted, and the five-point finite-difference Laplacian formula for the diffusion is considered. t_0 is also taken as 10 time units. Figure 4 shows the progress of eliminating spatiotemporal chaos with $A = 0.625$, $t_1 = 22$, and $t_2 = 25$. It is clear that anti-directional waves can be excited by feedback signals, and will meet soon with the original wave. The intersectional positions of the excited and unexcited states in Figure 4c are all in wave back, in this way, the excited domains will gradually diminish and finally disappear. Being different from eliminating spiral waves, the selected domains of t_1 and t_2 used to clear up spatiotemporal chaos are not contiguous due to the different development of many undeveloped spiral waves in spatiotemporal chaos. Figure 5 gives the optical values of t_1 and t_2 to eliminate spatiotemporal chaos for $A = 0.625$ and $A = 0.4375$ where t_1 and t_2 is forced to be selected from 20 to 60 time times. It can be seen that $t_1 < 31$ time units is needed to eliminate spatiotemporal chaos when $A = 0.625$; when $A = 0.4375$, $t_1 < 30$ time units is necessary. Generally, the optional range of t_1 and t_2 will diminish with the lessening of A. When $A = 0.0625$, the optional t_1 is only taken as 20 time units, and t_2 only taken as 31, 32, 33, 35 time units.

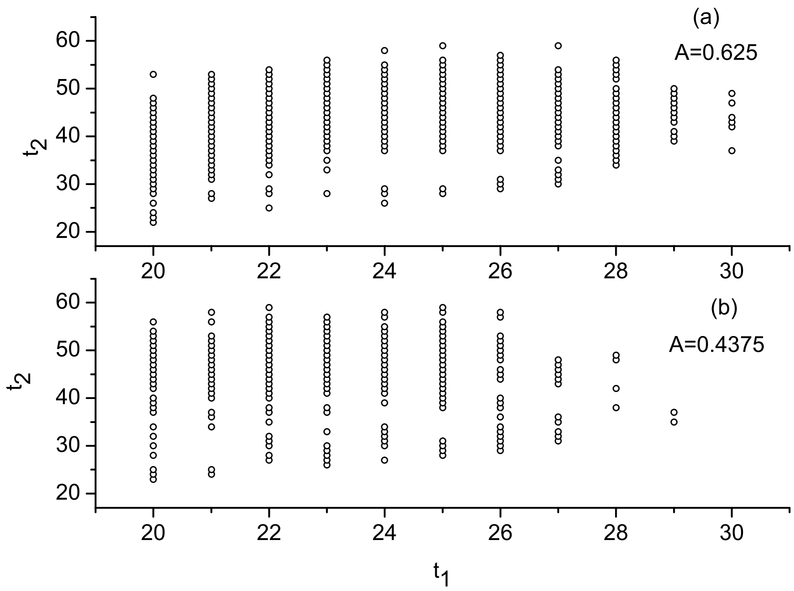

Fig. 5. The optional values of t_1 and t_2 eliminating spatiotemporal chaos. (a) $A = 0.625$; (b) $A = 0.4375$.

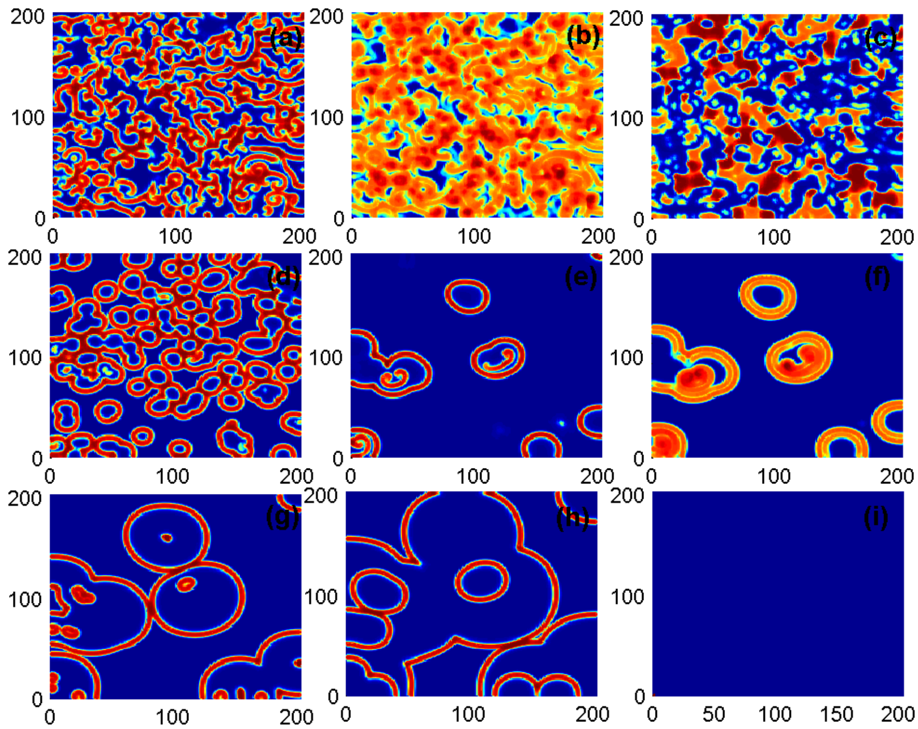

Fig. 6. Elimination of spatiotemporal chaos in the discrete Barkley model by twice feedbacks, where $A = 0.0625$. The first feedback signal is recorded at $t = 10$, then is injected into the system from $t_1 = 20$ to $t_2 = 29$. The second feedback signal is recorded at $t = 160$, then injected into the system from $t_1 = 170$ to $t_2 = 179$. (a) $t = 20$, the spatiotemporal chaos before the feedback is introduced; (b) $t = 29$, the first feedback has been injected; (c) $t = 49$, anti-directional wave starts to be formed; (d) $t = 93$; (e) $t = 160$, a majority of the tips have been removed, but a few of the tips are retained. At this time, the second feedback signal is recorded; (f) $t = 179$, the second feedback has been injected; (g) $t = 223$; (h) $t = 276$, the tips have been entirely removed; (i) $t = 662$, spatiotemporal chaos is completely eliminated.

For some t_1 and t_2, spatiotemporal chaos can not be completely eliminated via a single feedback, but where the majority of spiral tips can be removed and only a few tips are retained. Before the residual tips develop into spatiotemporal chaos, if the second feedback is injected into the system, spatiotemporal chaos can be completely eliminated. Sometimes, more tips are retained after the first feedback, and there are still a few residual tips via the second feedback, then more than twice feedbacks are needed. Figure 6 shows the progress eliminating spatiotemporal chaos by double feedbacks with $A = 0.0625$. From the above discussions, only when t_1 is taken as 20 time units and t_2 taken as 31, 32, 33, 35 time units between 20 and 60 time units,spatiotemporal chaos can be once eliminated.

Here $t_1 = 20$ time units and $t_2 = 29$ time units are taken, with which spatiotemporal chaos cannot be completely eliminated by the first feedback and only a few tips are retained (see Fig. 6f). We record again feedback signals at 160 time units, then inject them into the system between $t_1 = 170$ and $t_2 = 179$ time units, finally spatiotemporal chaos can be completely eliminated.

5 Conclusion and discuss

Spiral wave and spatiotemporal chaos after spontaneous breakup are harmful and should be eliminated in many real systems. In order to keep the original system undistroyed, weak signals and rapidly eliminating progress are neccessary. In this letter we present a scheme of eliminating spiral waves and spatiotemporal chaos by the feedback signal. With this scheme the tips can be simultaneously removed together with the body of spiral waves. So the elimination is generally speedy, which is difficult for some external force methods, At the same time, needed feedback signals may be very weak. All these characters are favored in real applications.

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